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Paper

ANALYSIS OF FLOW INSIDE THE FOCUSING TUBE OF THE ABRASIVE WATER JET CUTTING HEAD

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ABSTRACT

To make the analysis of flow in the focusing tube of abrasive water jet cutting head, the calculation of interval of the exit velocities of water jet from the focusing tube into the free space is necessary. The calculation of longitudinal velocity in the axis of water jet at the exit from the focusing tube, the calculation of expansion velocity of water jet at its divergence in the air and the calculation of mean velocity of water jet at the end of the focusing tube are performed. For the calculations, it was necessary to determine a radius of jet at its exit from the focusing tube. The analysis of longitudinal velocity in the focusing tube of abrasive water jet cutting head serves to the determination of a negative pressure in the mixing chamber of the cutting head. The value of a negative pressure is one of the boundary conditions of mathematical modelling of water jet flow in the cutting head.

1. INTRODUCTION

The analysis of a process of mixing of the liquid with air and particles of materials inside the cutting head is of importance to improvement in the efficiency of injection abrasive liquid jet. The mathematical description of liquid jet flow through the mixing chamber and the focusing tube can be done on the basis of knowledge of mass or volume concentration of air in the jet and the determination of influence of aeration of the jet on its velocity profile or on the mean velocity of the jet. The analysis of a longitudinal velocity in the focusing tube of abrasive water jet cutting head is of importance not only to the mathematical description of the phenomenon, but also to the determination of pressure of air inside the mixing chamber of equipment concerned.

At each point of stream, a fluctuating quantity, e.g. velocity, can be decomposed into two components, the mean (time-smoothed) velocity and the fluctuation velocity the magnitude and direction of which changes very quickly in time. The mean velocity is the average value of instantaneous velocity in the time interval, which is great enough with regard to the time of turbulent oscillations in fluctuation velocity. In the corresponding time interval, the mean value of fluctuation velocity is equal to zero. Similar considerations hold also true for other fluctuating quantities, such as pressure, density, and others. Although the time mean values of fluctuating components of quantities are equal to zero, the mean time value changes along the cross sectional area of flow of tubes. If we take, in the course of dynamical calculations, their mean value along the cross sectional area of flow into account, then the turbulent flow may be considered to be one-dimensional steady flow of liquid and relationships derived for this kind of flow can be used.

2. THE INTERVAL OF EXIT VELOCITIES OF WATER JET FROM THE FOCUSING TUBE TO THE FREE SPACE

In the cutting head, the water jet used for cutting of materials passes, after exiting the nozzle, through the mixing chamber and the focusing tube (Figure 1).

As known, a pure water jet or abrasive water jet is used for material cutting. When an abrasive is used, it is mixed with the water jet in the mixing chamber. In the following considerations and calculations, we shall be concerned only with a pure water jet passing the system mixing chamber – focusing tube. Calculations are executed on the following conditions:

- the liquid is incompressible,
- the liquid remains in the jet; it does not accumulate in the mixing chamber,
- the jet is not aerated.

In the course of calculations, we take the turbulent flow of water in the jet as a steady flow with the flow field characterised by the mean time values of fluctuating quantities (Kolář et. al. 1983).

At the beginning of calculations, we shall determine the interval of mean velocities at which the liquid can move at the exit from the focusing tube. From the equation of continuity it follows that

$$v_1 = \frac{S_0 v_0}{S_1} = \frac{\pi\left(\frac{d_0}{2}\right)^2 v_0}{\pi\left(\frac{d_1}{2}\right)^2}, \tag{1}$$

where S_0 is the cross-section of the nozzle at the entry into the mixing chamber, S_1 is the exit cross-section of the focusing tube, d_0 is the diameter of the nozzle, d_1 is the diameter of the focusing tube and v_0 is the mean velocity of water jet at the entry into the mixing chamber.

The value of velocity is the maximum if the jet does not extend radially from the axis, which is, with reference to the existence of radial components of velocity, impossible. The value of velocity is minimum if the jet expands in the radial direction in relation to the axis of flow so that its cross-section at the exit from the focusing tube will be identical with the cross-section of flow at the exit of focusing tube.

Specific calculations are performed for cutting head given by the following parameters and values of quantities characterising the condition of water:

- water nozzle at the entry into the mixing chamber: (index 0), $d_0 = 0.25\ mm$ ($r_0 = 0.125\ mm$),
- mixing chamber: (index s), $l_s = 13\ mm$, $d_s = 7.14\ mm$,
- focusing tube: (index 1), $l_1 = 76\ mm$, $d_1 = 1.02\ mm$ ($r_1 = 0.51\ mm$),

where l is a relevant length and d is a relevant diameter of respective parts of the cutting head.

The temperature t, water density ρ, pressure p_0 and mean velocity v_0 of water jet at the entry into the mixing chamber are expected to have the following values: $t = 20°C$, $\rho = 998\ kg.m^{-3}$, $p_0 = 400\ MPa$, $v_0 = 651\ m.s^{-1}$. For the cutting head set like that, the mean exit velocity of the liquid jet outflowing the focusing tube is valid $v_1 \in \langle 39, 651 \rangle\ m.s^{-1}$.

3. THE VALUE OF LONGITUDINAL VELOCITY IN THE AXIS OF WATER JET AT THE EXIT FROM THE FOCUSING TUBE

The water jet moving in the air begins expanding after overcoming a certain distance from the nozzle, called initial section. Along the initial section the velocity in the jet axis is uniform. Behind this section, the velocity begins to diminish as a result of jet expansion. The length of initial section l_p is given by the relationship of A. J. Milovič [2]

$$l_p = 145 d_0 . \tag{2}$$

The longitudinal maximum velocity v_1 along the axis of water jet moving in the air is, within its compact part behind the initial section, given by relationship (Agroskin et. al. 1955)

$$v_l = \frac{145 v_0 d_0}{l}, \tag{3}$$

where l is the distance measured along the jet axis from the end of initial section.

For the cutting head with the above-presented parameters, we shall get the length of the initial section $l_p = 36\ mm$ and the velocity within the section $v_s = 651\ m.s^{-1}$ after inserting into these relationships. At the distance $l_k = 53\ mm$ from the end of the water nozzle, the velocity of water begins to decrease according to relationship (3). Hence we shall determine the value of velocity in the axis of jet at the end of the focusing tube; the velocity reaches the value $v_1 = 265\ m.s^{-1}$.

4. THE EXPANSION VELOCITY OF WATER JET AT ITS DIVERGENCE IN THE AIR

In the radially expanding part of water jet behind the initial section, a decrease in the velocity of water jet occurs.

For the calculation of expansion velocity v_e in the direction perpendicular to the axis of water jet, we shall use a relationship derived by Hlaváč (Hlaváč et. al. 1999a)

$$v_e = \left[\frac{2}{\rho_0} \left(p_0 - \frac{1}{2} \rho_0 v_{0u}^2 - p_{at} \right) \right]^{\frac{1}{2}}, \tag{4}$$

where p_0 is the pressure of liquid before the nozzle, ρ_0 is the density of liquid under normal conditions, v_{0u} is the velocity of water jet at the entry into the mixing chamber taken approximately as equal to v_0.

In a case of the set cutting head, we shall obtain for the value of temperature $t = 20°C$ and the density of water $\rho = 998\ kg.m^{-3}$ the calculated value of expansion velocity is approximately $v_e = 615\ m.s^{-1}$. The expansion of the jet at the nozzle outlet causes a small increase of water jet diameter. Nevertheless, the expansion attenuates very quickly.

5. RADIUS OF JET AT THE OUTLET FROM THE FOCUSING TUBE

We shall determine the radius of jet y_1 as a sum of its radius r_0 at the entry into the mixing chamber and the distance y, i.e. the distance travelled by the marginal part of jet in the radial direction perpendicularly to the axis of the tube per time needed by the jet to travel the distance from the entry into the mixing chamber to the exit from the focusing tube

$$y_1 = r_0 + y = r_0 + v_e t, \tag{5}$$

where t is the time needed by the jet to travel the total internal path in the cutting head.

The total internal path l is taken as a sum of the length of mixing chamber l_s and the length of focusing tube l_1. We shall divide the internal path into two sections, i.e. the initial section and the section in which the velocity of jet diminishes.

We shall designate:

$\bar{v}_1$ …the mean velocity of jet along the initial section,
$\bar{v}_2$ …the mean velocity of jet along the section with the decreasing velocity.

In a case of the initial section, the velocity along the path is uniform. With the other section, we shall determine the mean value of velocity from equation (3) by replacing distance l by variable x. By subsequent integration along the length of section we shall obtain

$$\bar{v}_2 = \frac{1}{l_x}\int_{l_p}^{l}\frac{145 v_0 d_0}{x}\,dx = \frac{1}{l_x}145 v_0 d_0 \ln\frac{l}{l_p} \ , \qquad (6)$$

where $l_x = l - l_p$.

For the set parameters of cutting head, the following then holds true: $\bar{v}_1 = 651\, m.s^{-1}$, $\bar{v}_2 = 403\, m.s^{-1}$.

The presented time is given by a sum of times needed for covering the two above-mentioned sections, i.e.

$$t = t_1 + t_2 = \frac{l_p}{\bar{v}_1} + \frac{l_k}{\bar{v}_2} \quad . \qquad (7)$$

For the set cutting head we shall obtain almost $t = 1.87 * 10^{-4}\ s$.

After putting down into relationship (5) we shall obtain the value of radius of water jet y_1 at the exit from the focusing chamber. For the set values of cutting head it has the value about $y_1 = 115\ mm$. This result clearly presents that the classical theories presented for low velocity jets are unusable for high-velocity water jets.

Therefore we used the theory presented in Hlaváč et. al. (1999a) and calculated the radius of the jet at the end of the focusing tube, i.e. in the distance $l = 89\ mm$ from the water nozzle outlet. The radius is then $y_1 = 0.324\ mm$. The width of the gap $r = r_1 - y_1$ between the focusing tube and the water jet at the exit from cutting head is then $r = 0.186\ mm$.

6. THE MEAN VALUE OF THE VELOCITY OF WATER JET AT THE END OF THE FOCUSING TUBE

The equation of continuity will be written for the cross-section of jet at its entry into the mixing chamber and the exit cross-section of jet at its exit from the focusing tube as follows

$$S_0 \overline{v}_0 = S_{y_1} \overline{v}_1 , \tag{8}$$

where $S_0 = 0.049\ mm^2$ is the cross-section of jet at the entry into the mixing chamber, $\overline{v}_0 = 651\ m.s^{-1}$ is the mean velocity of water jet at the entry into the mixing chamber, $S_{y1} = 0.330\ mm^2$ is the cross-section of water jet at the exit from the focusing tube, v_1 is the mean value of velocity of jet in its exit cross-section. With the set cutting head, the mean velocity of water jet at the exit from the focusing tube is expected to be $\overline{v}_l = 97\ m.s^{-1}$ according to the equation (8).

The theoretical model published in Hlaváč et. al. (1999b) yields for the length of the jet core with the assigned values of the nozzle diameter and water pressure approximately $l_k = 61.5\ mm$. The average velocity at the water nozzle outlet is just $\overline{v}_0 = 646\ m.s^{-1}$. The axis velocity at the end of the focusing tube is close to the value $v_{2o} = 571\ m.s^{-1}$ and the jet diameter is about $r_l = 0.324\ mm$. Those parameters yield the jet cross-section nearly $S_{y1} = 0.330\ mm^2$ by the end of the focusing tube and the average outlet velocity of the water jet is almost $v_2 = 96\ m.s^{-1}$. Simultaneously, the average velocity determined from the velocity profile of the same model and the effective value of the jet cross-section is close to the value $v_1 = 94\ m.s^{-1}$, i.e. almost the same as the one determined from the equation of continuity.

7. CONCLUSION

The analysis of longitudinal velocity in the focusing tube of abrasive water jet cutting head consists in the determination of minimum velocity, at which the jet can exit from the focusing tube into free space, in the determination of initial section of the jet and its velocity at the end of the focusing tube, in the determination of average velocity along the whole internal path of the jet, in the determination of expansion velocity of the jet at its divergence in the air and in the calculation of mean value of velocity of water at the end of the focusing tube.

The performed analysis of longitudinal velocity in the focusing tube of abrasive water jet cutting head is necessary for the determination of negative pressure in the mixing chamber of cutting head, which is one of boundary conditions of mathematical modelling the flow of water jet through the cutting head.

8. ACKNOWLEDGEMENTS

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10. GRAPHICS

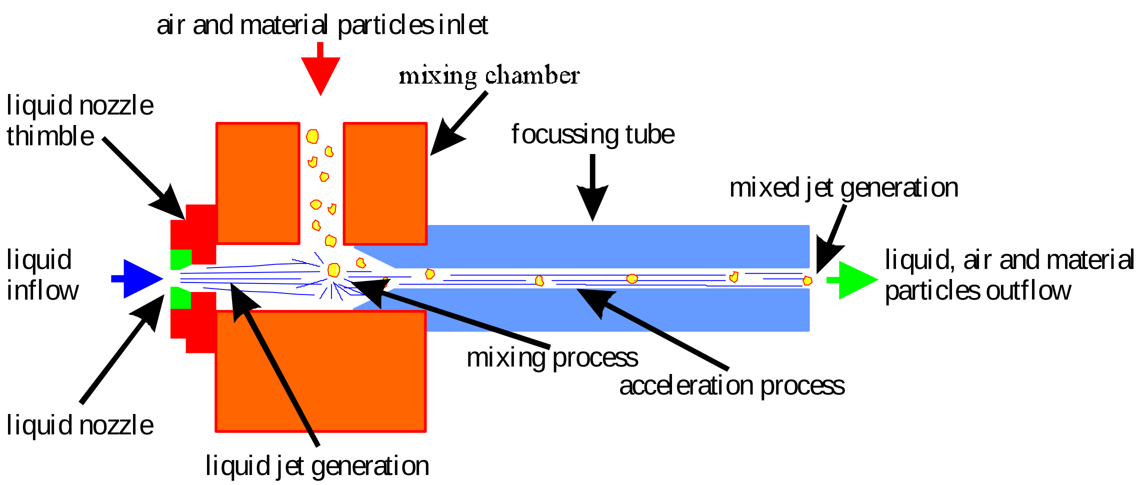

Figure 1. A diagrammatical section through the cutting head with the representation of processes leading to the generation of abrasive liquid jet at its exit.