

## **ANALYSIS OF THE ABRASIVE WATERJET DRILLING PROCESS**

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### **ABSTRACT**

Since its inception a little over two decades ago, the Abrasive Waterjet (AWJ) process has gained immense popularity owing to the numerous advantages offered by this process like absence of heat-affected zone and no residual stresses. These days this process is being applied into the drilling of hard-to-cut materials. It is especially useful in applications like deep hole drilling. However in AWJ drilling of blind holes, the back-flow of the impacting jet and the standoff distance influences the shape of the hole drilled. This shape of the hole can be critical in applications where exact hole dimensions are required. The AWJ process parameters like pressure, flow rates etc also affect the dimensions of the hole as well as the time required drilling. The drilling time can be critical in applications where machining times are constraints. These effects and issues can be investigated through the mathematical modeling of the AWJ drilling process. Thus for a better understanding of the AWJ drilling process, a need exists to understand the models published so far to describe this process. This paper attempted to review briefly all the published models and critically evaluate them to highlight the advantages and the limitations of the existing models. Representative experimental data has been utilized as the common platform for evaluating all the models including the recently developed conical cavity model.

## 1. INTRODUCTION

The abrasive waterjet (AWJ) cutting process has gained immense popularity since its inception in the early 1980s, especially in the machining of hard-to-cut materials [1, 2]. The absence of heat-affected zones and residual stresses in the workpiece material has been a primary factor [3]. The process is capable of producing a product to finished or near-finished dimensions with minimal material loss.

Drilling is one of the most common machining processes, accounting for most of the material removed by all metal cutting processes. It involves creating holes of right circular cylindrical shape, traditionally by employing rigid twist drills. In deep-hole applications, removal of the chips and cooling of the cutting front are significant issues involved with traditional drilling operations. However, the AWJ drilling or piercing process involves impacting the target material with an abrasive-laden waterjet, directed normal to the target surface, to penetrate the material by erosion [4]. The process is continuous and clean; leaving no heat affected zones or residual stresses. Since both the eroded material and any generated heat can leave the cavity with the out-flowing slurry, the issues of chip removal and cutting front cooling are avoided.

The process of penetrating a material with a stationary AWJ can be broadly classified into three categories: piercing, trepanning, and drilling [1]. Piercing involves creating a hole through the entire thickness of the target material. Because the hole passes all the way through, the shape of the drilled cavity and the back-flow of the abrasive jet are not of much concern. Trepanning involves enlarging previously cut holes; thus, cavity shape and back-flow are again of less importance. The AWJ drilling process, however, involves creating blind holes, whose depth and internal shape may be difficult to control accurately. One problem is determining the time necessary to drill a hole down to a particular depth. Also, due to the nature of the abrasive jet and the mechanics of the erosion process, the jet may not necessarily produce straight-walled or non-tapered holes like traditional drills [5]. Hole taper may not be acceptable in applications where accurate dimensions are required. Non-tapered through-holes are possible in piercing applications, but generally require keeping the jet on for some time beyond that required for simple piercing. To mathematically express the shape of the drilled cavity is quite complex [5], owing to the nature of the numerous machining parameters involved and the complex interactions among them. So there is a need for accurate models of the AWJ drilling process, which can be used to determine optimal ranges for process parameters under any arbitrary set of conditions.

To better understand the AWJ drilling process, the goal of this research is to investigate the existing drilling models and evaluate how well the model predictions reported in the literature compare with experimentally obtained data. Evaluation of the published models will show how well they are able to predict the depth of the drilled cavity. To accomplish the stated objective, we utilized the extensive amounts of data collected through experimental work carried out in the High-Pressure Waterjet Laboratory at the University of Washington. This extensive data set gives us a broad platform for evaluating the existing models.

## 2.1 Hole Profile Observations

In AWJ drilling, the hole produced is substantially wider than the jet stream, due mostly to the additional wall erosion resulting from the forceful upward ejection of the jet out of the blind cavity. A comprehensive model of the process would have to account for both the primary erosion front behavior, determining the hole depth and penetration rate, as well as the backflow phenomena affecting final diameter and profile shape (Figure 1).

The annular backflow region surrounding the incoming jet presents a highly turbulent and chaotic flow situation. The ejected stream is a churning mixture of water and air, laden with both shattered and intact abrasive particles as well as fragments of removed material. It interacts with both the incoming flow and the irregular, continuously evolving cavity surface. Modeling this situation analytically is thus extremely difficult.

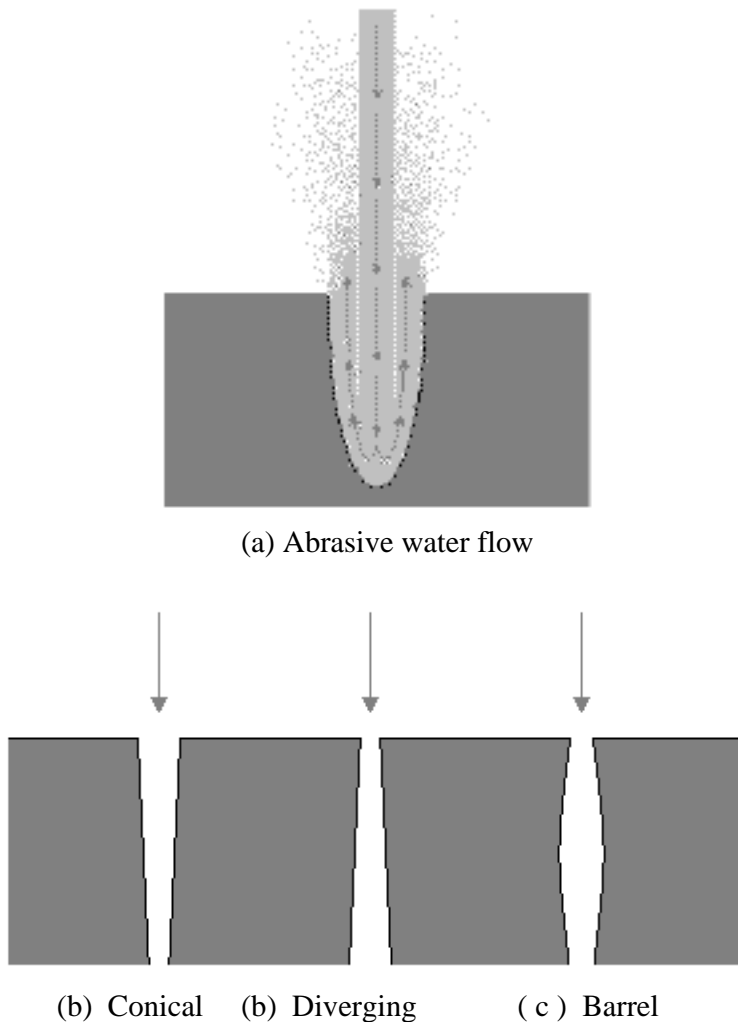


Figure 1 Typical AWJ drilled hole and hole configurations

### 3. REVIEW OF ABRASIVE WATERJET DRILLING MODELS

As with any process, the modeling of AWJ drilling can provide a better understanding of the mechanisms involved and the ways in which the various process parameters interact to effect overall performance. However, AWJ process modeling poses many challenges owing to the large number of parameters, the complex interactions among them, and the difficulty of describing some of these interactions in analytical forms [4, 7]. Not much work has been reported in the literature concerning AWJ drilling, and very few models of the drilling process exist. All the models reported to date in the literature can be classified into the following category [1]: physical (or phenomenological) models, regression or empirical models, simulation models, and analytical models. Almost all of the published models are formulated around an expression relating the volume removed in the erosion process to the kinetic energy expended in removing that volume.

#### 3.1 Physical Models

The energy of the jet impinging normally on the target material surface is consumed in removing some volume of the material. This results in decreasing penetration rate with increasing depth, which continues until a limiting depth, termed the maximum depth, is reached [5]. A plot of penetration depth vs. exposure time generally takes on the shape of an exponential curve, and many researchers use this to model the process. Such models generally employ the following relation [3, 5]:

$$h(t) = h_{\max} (1 - e^{-Ct}) \quad (1)$$

Where  $C$  is a parameter sensitive to jet properties like pressure, velocity, abrasive size, etc. The main limitation of such models is that the maximum depth of penetration has to be ascertained in advance. The equation (1) can best be utilized for piercing where the material thickness to be penetrated is already known.

Another physical model was recently developed by Hashish based on Bitter's model for volumetric removal rates in ductile materials [1]. It is an interesting model that takes the abrasive size into account analytically. The assumptions are:

1. Abrasive flow rate is uniformly spread across the jet.
2. The volume removal rate and jet kinetic energy is related through a material strength parameter.
3. Abrasive particles decelerate through a simple viscous drag force.

The final expression for the depth of penetration is:

$$h = \frac{1}{2k_2} \ln(1 + 2k_1 k_2 U^2 t) \quad (2)$$

where

$$k_1 = \frac{\dot{m}_a}{2\pi R^2 \sigma_f}$$

and

$$k_2 = \frac{C_D \rho_s A_p}{2\dot{m}_a}.$$

### 3.2 Regression or Empirical Models

Zeng and Muñoz presented a regression model for estimation of the time required piercing a hole of depth  $h$  [8]. The time is given by

$$t_p = \frac{c_0 h^{c_1} d_0^{c_2}}{N_{mP} P^{c_3} d_F^{c_4} \dot{m}_a^{c_5}} \quad (3)$$

Where the coefficients  $c_0$  to  $c_5$  can be determined from regression analysis. The model accounts for the target material characteristics through the machinability number parameter  $N_{mP}$ .

### 3.3 Simulation Models

These models use a finite-element approach to simulate the drilling process, and yield good results when the maximum depth has been ascertained beforehand. Yong and Kovacevic [6,9] developed a model based on simulation of the process by considering the chaotic erosion history of millions of particles (Figure 2). The constituting equation for the depth of erosion caused by a single particle was extended and applied to a large number of particles, incorporating their individual erosion histories.

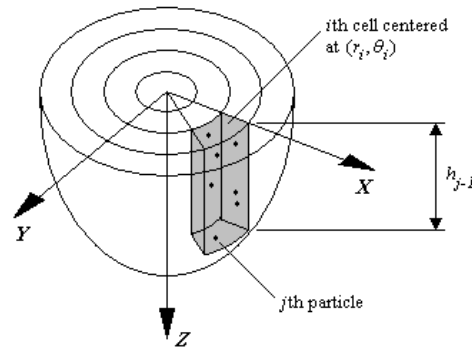


Figure 2 Memory cell model of the erosion cavity

The final constitutive relation for the penetration depth of a particle, in dimensionless form, is:

$$\delta \bar{h}_j = \frac{1}{\bar{h}_{j-1}^2 + 1} [V_j(r_i, \theta_i)]^{2\bar{h}_{j-1} + 2}. \quad (4)$$

Here,  $V_j = v_j/v_{\max}$  is the dimensionless velocity of the  $j$ th particle as it leaves the nozzle. The left-hand-side is the dimensionless penetration depth of the  $j$ th particle:

$$\delta \bar{h}_j = \frac{\delta h_j}{l}$$

where  $\delta h_j$  is the actual penetration depth and  $l > 0$  is a constant to be determined experimentally. Similarly,

$$\bar{h}_{j-1} = \frac{h_{j-1}}{l}$$

is the overall penetration depth achieved by the  $j-1$  particles that impact prior to the  $j$ th particle. These quantities all refer to particles passing through the  $i$ th cell, with center coordinates  $(r_i, \theta_i)$ . The incoming jet radius is taken as a unit distance, so  $0 \leq r_i \leq 1$  for particles within the main jet stream. Two forms of the velocity profile  $V_j$  are used:

$$V_j = 1 - r_i^2$$

for laminar flow, and

$$V_j = (1 - r_i^2)^{1/7}$$

for turbulent flow. Inserting these in equation (4) gives two different versions of the constitutive relation, which yield somewhat different predictions for the penetration depth and cavity shape.

### 3.4 Semi-Empirical Models

These models are developed from analytical principles, but due to the quantity and nature of the parameters and their interactions, have to rely on some empirical constants for solution and validation. These models are based on Bitter's model for predicting volume removal rates in ductile materials.

Capello and Groppetti developed a semi-empirical model for predicting the shape and size of the kerf generated by AWJ machining [7]. The fundamental model is the same for both cutting and piercing, except that for piercing the traverse component would not be present. The assumptions of the model are:

1. The relationship between the volume removed and the kinetic energy imparted is of first order.
2. Water plays a limited role in the erosion mechanism.
3. Power distribution within the jet is non-uniform and symmetric about the jet axis.
4. Iso-energetic curves in the x-y plane are circumferences centered around the jet axis.

The model is based on a simple proportionality between an elemental volume of removed material and the amount of abrasive flow kinetic energy consumed in eroding that volume. The energy considered available for erosion is only a portion of the total jet energy; it is found by scaling the initial jet energy through a decreasing function of  $h$ , which accounts for energy dissipation within the kerf and the loss of erosion efficiency due to jet deflection. Only the abrasive mass flow rate is considered in calculating the initial jet energy. The model also accounts for the non-uniform energy distribution within the jet, through the use of a *Spatial Distribution of Power Density* function (SDPD). This is chosen such that the energy density is maximum along the jet axis and zero at the periphery.

The final expression for the depth penetrated at any time  $t$  is

$$h(\rho, t) = 1 + k_f (\gamma + 1) \frac{\dot{m}_a v_j^2}{I(\alpha, \beta)} (1 - \rho^\alpha)^\beta t^{\frac{1}{\gamma + 1}} - 1 \quad (5)$$

where

$$I(\alpha, \beta) = \int_A (1 - \rho^\alpha)^\beta dA$$

arises from integration of the SDPD over the erosion area  $A$ . Here  $\rho$  is the normalized radial distance from the jet axis (i.e., at a distance  $r$  from the axis,  $\rho = r / r_j$ , where  $r_j$  is the jet radius). The maximum depth thus occurs at the center, where  $\rho = 0$ . The empirical nature of the model is due to the four parameters,  $k_f$ ,  $\gamma$ ,  $\alpha$ , and  $\beta$ , which are obtained by regression analysis.

This model, like others based on similar considerations, does not account for the effects of jet backflow. It relates volume removal to the kinetic energy of the abrasive particles upon initial impact. The erosion effects of pure water have also been neglected; however, studies at the University of Washington have shown that pure water does aid in the erosion process.

### 3.5 Analytical Models

The Raju-Ramulu model [2, 10] is the only one presented here that attempts to proceed analytically from the principles of fluid mechanics. Some sources classify it as a fully analytical model, though it incorporates a few constants, which must be determined experimentally. The model is based on the idea of momentum and energy conservation between the incoming and outgoing jet streams within the eroded cavity. The erosion rate is related to the difference in kinetic energy between these two streams. The following assumptions are made to simplify the model formulation:

1. The piercing process is quasi-static in time.
2. The cavity is cylindrical and frozen in time, with depth  $h$  and radius  $R^*$ .
3. The standoff distance is small.
4. Boundary layer effects at the target surface are negligible.
5. The incoming and outgoing streams have uniform mean velocity profiles.

The model assumes a cylindrical cavity, having dimensions  $R^*$  and  $h$  at a particular instant in time, as shown in Figure 3. The origin of the coordinate axes is taken as the center of the initial jet impingement area on the intact target surface, with the  $y$ -axis positive in the direction of the advancing cavity floor. The velocities of the incoming and outgoing streams,  $v_j(y)$  and  $v_f(y)$ , are estimated based on the drag forces resulting from interactions between the two streams. The velocity change yields a net change in kinetic energy at the point of maximum depth of the drilled hole, at  $y = h$ , and this is related to the material removal rate.

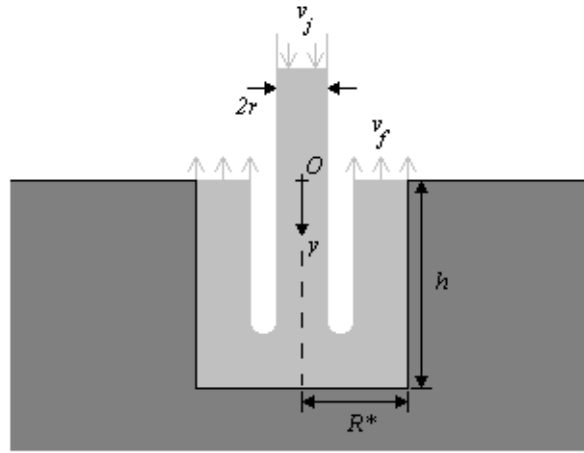


Figure 3 AWJ drilling of a cylindrical cavity

The final model equation for the penetration rate is:

$$\frac{dh}{dt} = k_3 \frac{1}{k_1 h + a} - \frac{dh}{dt}^2 \quad (6)$$

where

$$k_1 = \frac{\pi r C_D \rho_s}{\dot{m}_a + \dot{m}_w} \left( 2 + \frac{R^*}{r} \right)^2$$

and



$$k_3 = \frac{5\xi\dot{m}_a}{6\pi R^{*2}}$$

Here  $\xi$  is an empirical constant, termed the *inverse specific erosion energy*. It can be interpreted as the inverse of the energy consumed in eroding a unit volume of material from the bottom of the cavity.

The cylindrical cavity assumption used in developing the fluid flow relations agrees approximately with observed cavity shapes. However, a modified “conical-cavity” version of this model has also been developed that accounts for the taper seen in actual AWJ drilled holes [11].

$$\frac{dh}{dt} = \frac{k_3}{(R^* + h \tan\theta)(R^* + 3h \tan\theta)} \frac{1}{a + k_1 h} \quad (7)$$

$$k_1 = \frac{\pi r C_D \rho_s}{\dot{m}_a + \dot{m}_w} \left( 2 + \frac{R^*}{r \cos\theta} \right)^2$$

$$k_3 = \frac{5\xi\dot{m}_a}{6\pi R^{*2}}$$

where  $\theta$  is the conical cavity half-angle.

The transcendental form of equations (6 and 7) and the need to calibrate certain parameters experimentally complicate solution of the model. Nevertheless, the development approach is chiefly analytical, with some effort made to account for the effects of the back-flowing jet and the role of water in the erosion process.

#### 4 RESULTS AND DISCUSSION

In AWJ drilling process, the depth penetrated increases with a decreasing rate until it reaches a limiting value termed as the maximum depth drilled. Beyond this depth, the changes in the depth of the hole are negligible. These phenomena can be observed regardless of the material or the process parameters. Figure 4 presents the drilled depths for a variety of materials and machining conditions plotted using the experimental data from the References [5, 10, 12]. Clearly the energy dissipated in generating the hole depends on the material properties. Figure 5 shows the AWJ depth of penetration in polycarbonate materials with conical cavity model [11] and the effects observed have been summarized in the Table 1. The figure gives us an understanding of the nature of the depths drilled over time. Before studying the empirical constants in the model, we first need to understand the effects of process parameters and empirical constants on the depths predicted by the model. Understanding these effects would aid us in better selection of process parameters for a particular drilling application. The process parameter varied in the experimentation was supply pressure; abrasive flow rate and abrasive sizes.

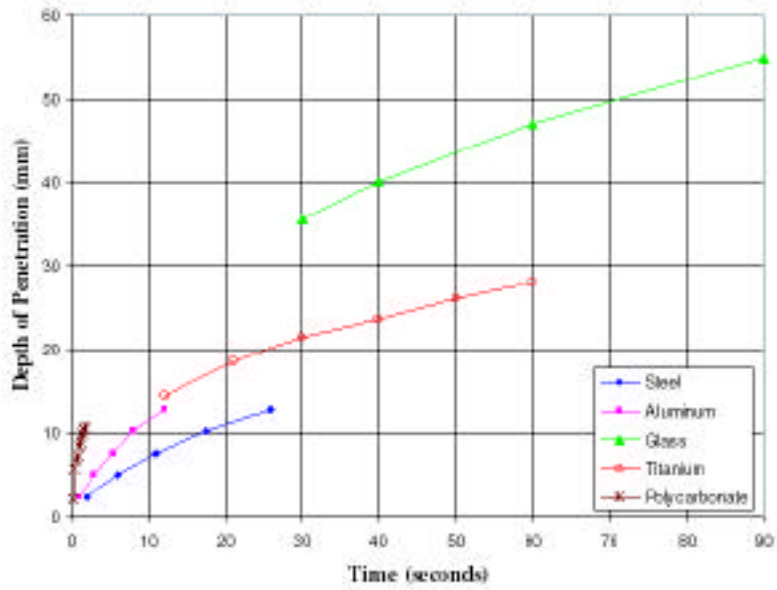


Figure 4. Depth of Penetration against Time for a Variety of Materials

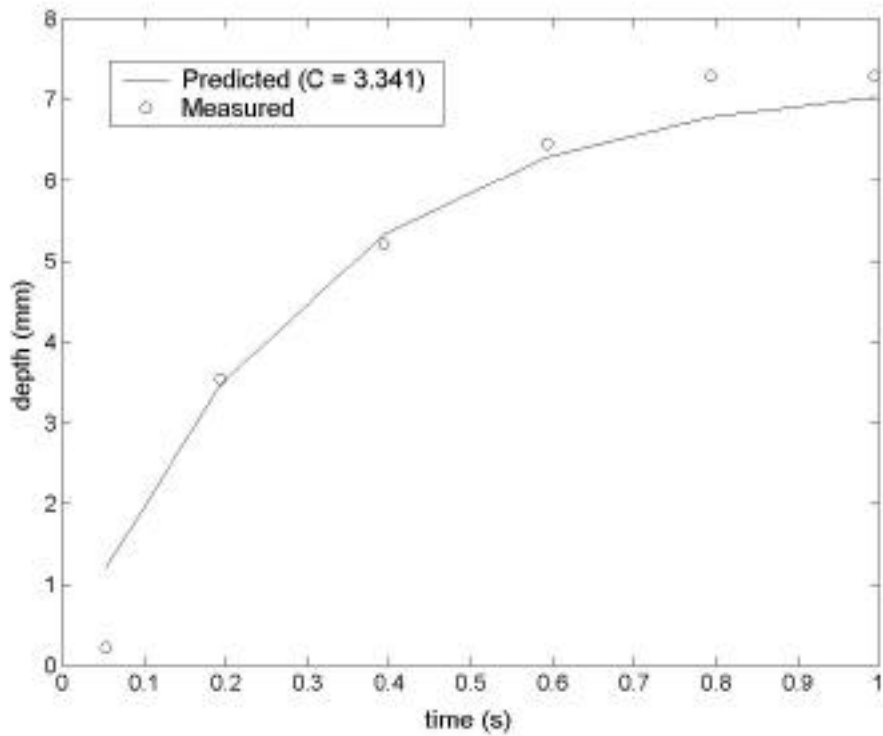


Figure 5. Conical Cavity model depth prediction in Polycarbonate material

Table 1 Effect of Process Parameters and Empirical Constants on Predicted Depths

		Predicted Depth
Supply Pressure	↑	↑
Abrasive Flow Rate	↑	↑
Abrasive Size	↑	↓
$k_1$	↑	↓
$\xi$	↑	↑

Figure 6 presents the plot of the depths of penetration predicted by both the cylindrical as well as the conical cavity models against the experimental depths of penetration [11,12]. Forty-two experimental data sets were available for comparison and all these data sets have been used here. The values of the empirical constants used in the cylindrical and the conical cavity model for predicting the depths have been presented in Ref [11]. The plots show generally good correlation between the experimental and predicted depth especially for the conical cavity model.

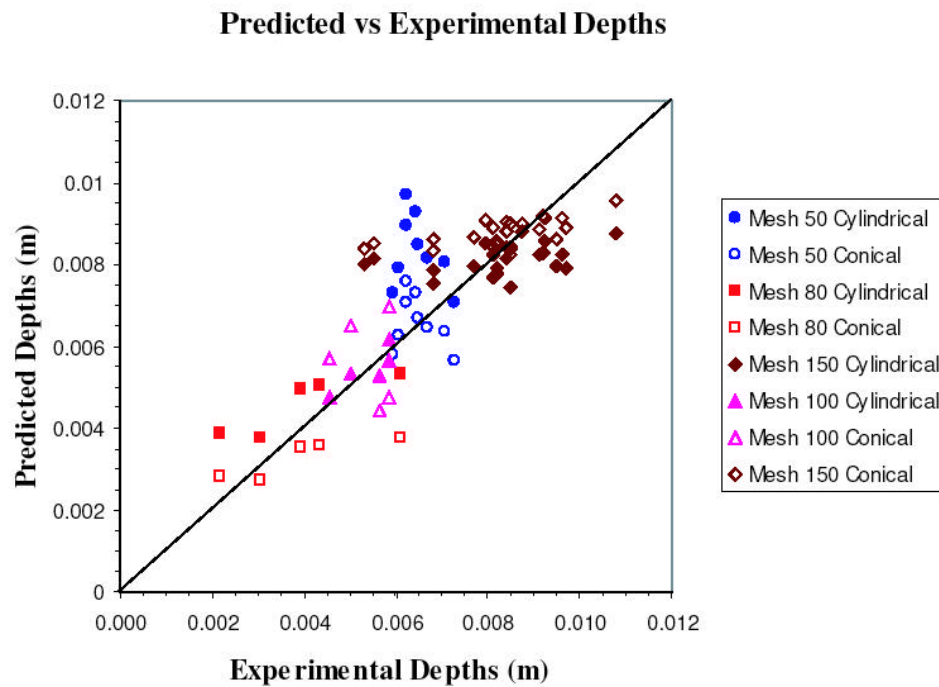


Figure 6 Correlation between the experimental and the predicted depths for the entire experimental data sets

To critically evaluate the published models, it is necessary to test these models under similar conditions. In this research all these models were tested using the same experimental data

collected here in University of Washington. Figure 7 shows the drilling depths predicted by these models for one such case. It may be noted that the Kovacevic's chaotic erosion history model predicts the same depth as the experimental depths because the model can be fully solved only on knowing the maximum experimental depth. Thus, it's a simulation model that gives us the depths drilled over time. But all the models overpredict during early time interval. A look at the following figure shows that as the time increases the changes in depths decreases rapidly and towards the end it's very small. However, all the models predict that the depth goes on increasing though at a very small rate. But of all the models, the conical cavity model seems to fit the model the best. Though the relative error between the cylindrical cavity model and the conical cavity model is not very large. On observing the shape of the curves and the experimental data, we may observe that the cylindrical and conical cavity models follow the experimental data better than the other models. The reasons for this is that these two models are based fluid dynamics principles and are analytical models as well as they having accounted for the backflow of the impinging jet. A comparison for the predicted depths cannot be done with the Kovacevic model, as it's a simulation model dependent on the maximum depth.

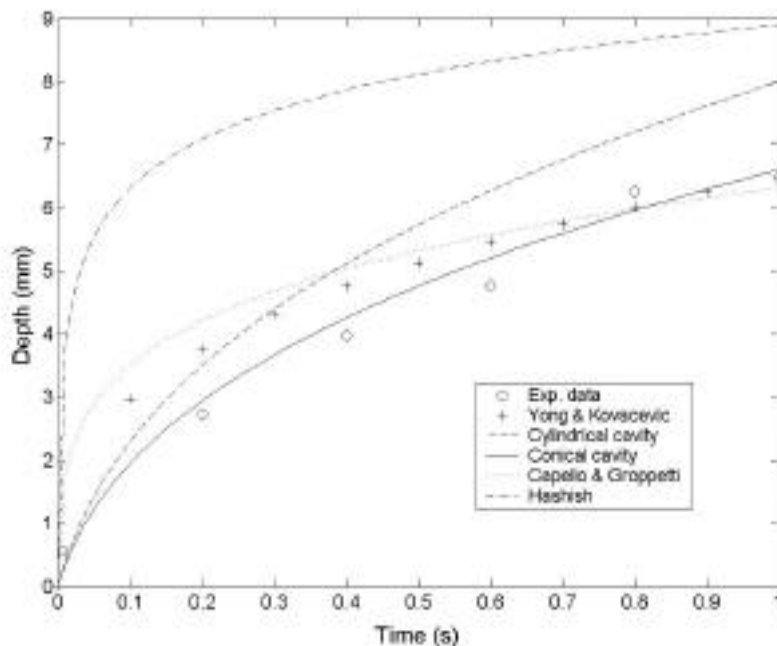


Figure 7: Depths of penetration predicted by existing models under similar conditions

The nature of the depth of penetration versus time certainly bears a resemblance to a logarithmic curve. The expression for the final depth in Hashish's model contains a logarithmic term. But due to this logarithmic term the model over-predicts the drilled depths quite a bit in the early stages.

To study the predictions by Kovacevic's Chaotic Erosion History Model and the Conical Cavity Model; the experimental data utilized by Kovacevic to test their model was used [6,8]. The predictions by the Conical Cavity Model also were in as good correlation as the Kovacevic's Model as shown in Figure 8. However, Kovacevic model's prediction was based on the experimentally knowing the maximum depth whereas in the conical model, we need not know

the experimental depths if the empirical constants are known. Thus, without knowing the experimental depths, we can obtain quite good predictions as shown by the following figure.

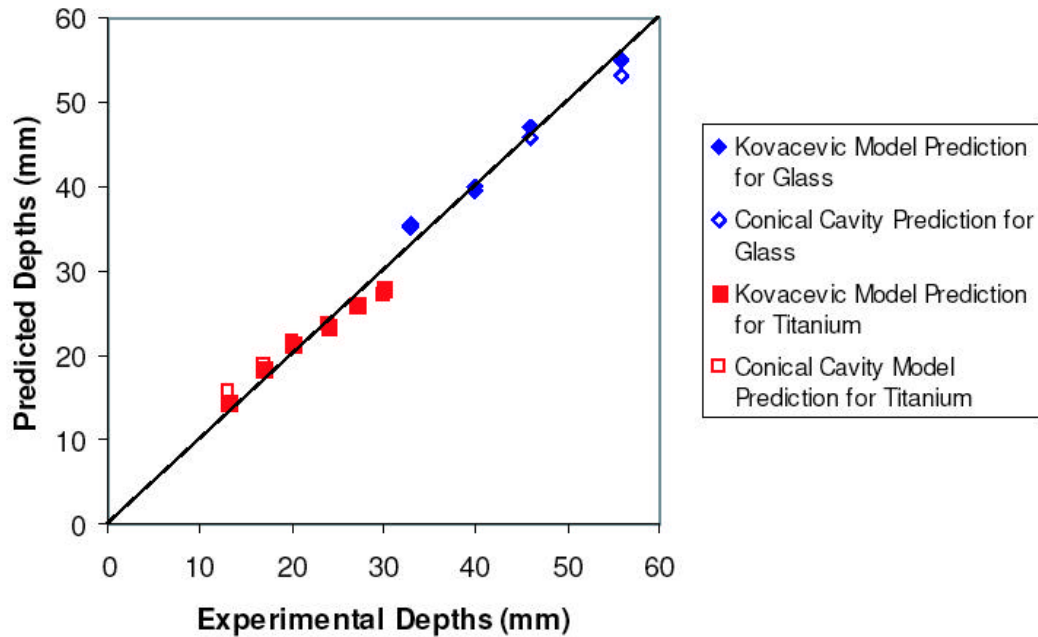


Figure 8. Correlation between Experimental and Predicted Depths for Kovacevic’s Model as well as Conical Cavity Model

Most of the models discussed in this review were based in one form or another on the conservation of the energy of the jet and its balance by the material removal rate. Yong and Kovacevic’s model uses a constitutive equation for the depth of penetration that is a modified form of the energy balance expression. Capello and Groppetti have equated the elemental volume removed to the elemental kinetic energy imparted by the abrasive particle to derive a model based on the spatial distribution of the power density within the jet. Regardless of the end form, all these models are based on the balance between the kinetic energy supplied by the jet and the volume of the material removed. Except for the Raju-Ramulu model, none of the models have taken into account the backflow of the jet. It has been experimentally shown that the backflow of the jet has considerable impact on the shape of the cavity drilled and hence would help in better prediction of the drilling process. Therefore, Raju-Ramulu model is a novel approach as it takes into consideration the back- flow of the water. All the models reported in the literature so far have utilized the expression relating volume removal rate to the jet energy but none of the literature has shown an analytical approach to obtain that relation. Yong and Kovacevic’s model needs the maximum depth for better simulation of the process. It is better suited for piercing process where the thickness of the material is already known. Such models are not suited for cases where one needs to ascertain the maximum depth drilled for particular machining conditions or time taken to drill that. Summary of all the models and the final expressions for the models discussed here have been summarized in Table 2.

Table 2. Summary of AWJ Drilling or Piercing Models

Model		Remarks
Yong and Kovacevic's Chaotic Erosion History Model	<p>For laminar flow:</p> $\delta \bar{h}_j = \frac{1}{\bar{h}_{j-1}^2 + 1} (1 - r_i^2)^{0.2 \bar{h}_{j-1} + 2}$ <p>For turbulent flow:</p> $\delta \bar{h}_j = \frac{1}{\bar{h}_{j-1}^2 + 1} (1 - r_i)^{\frac{0.2 \bar{h}_{j-1} + 2}{7}}$	<ul style="list-style-type: none"> <li>• Back-flow of the jet not considered</li> <li>• Maximum depths are needed for solution</li> <li>• Independent of supply pressure and mesh size</li> </ul>
Capello and Groppetti's Semi-Empirical Model	$h(\rho, \tau) = 1 + k(1 + \gamma) \frac{\dot{m}_a V_j^2}{I(\alpha, \beta)} (1 - \rho^\alpha)^\beta \tau^{\frac{1}{1+\gamma}} - 1$ $I(\alpha, \beta) = \int_A (1 - \rho^\alpha)^\beta dA$	<ul style="list-style-type: none"> <li>• Back-flow of the jet not considered</li> <li>• The model constants, <math>k</math>, <math>\gamma</math>, <math>\alpha</math>, and <math>\beta</math>, need to be validated for solution</li> </ul>
Cylindrical Cavity Model	$\frac{dh}{dt} = k_3 \frac{1}{a + k_1 h} - \frac{dh}{dt}^2$ $k_1 = \frac{C_D \rho_s \left( 2 + \frac{R^*}{r} \right) (2\pi r)}{2 \dot{m}_a + \dot{m}_w}$ $k_3 = \frac{5 \xi \dot{m}_a}{6\pi R^2}$	<ul style="list-style-type: none"> <li>• Takes into account the backflow of the jet</li> <li>• Developed analytically on fluid dynamics principles</li> <li>• Cavity assumed to be cylindrical; experimentally verified to be nearly cylindrical</li> </ul>
Conical Cavity Model	$\frac{dh}{dt} = \frac{k_3}{(R^* + h \tan \theta)(R^* + 3h \tan \theta)} \frac{1}{a + k_1 h}^2$ $k_1 = \frac{C_D \rho_s \left( 2 + \frac{R}{r \cos \theta} \right) (2\pi r)}{2 \dot{m}_a + \dot{m}_w}$ $k_3 = \frac{5 \xi \dot{m}_a}{6\pi}$	<ul style="list-style-type: none"> <li>• Introduces concavity to the Raju-Ramulu model</li> <li>• Cavity drilled is conical in shape</li> </ul>
Hashish's Model	$h = \frac{1}{2k_2} \ln(1 + 2k_1 k_2 U^2 t)$ $k_1 = \frac{\dot{m}_a}{2\pi R^2 \sigma_f}$ $k_2 = \frac{C_D \rho_s A_p}{2\dot{m}_a} = \frac{3C_D}{4sd_p}$	<ul style="list-style-type: none"> <li>• Back-flow of the jet not considered</li> <li>• Still needs to be numerically verified</li> <li>• Nature of material strength parameter has to be ascertained</li> </ul>

## 6. SUMMARY AND CONCLUSIONS

Critical evaluation of all the published models was done and compared with the conical and cylindrical cavity models. Both cylindrical and conical cavity models were found to be predicting better results than the published models.

This research forces some questions of itself that need to be looked into, and which serve as pointers for further research into this area. They are:

1. Can we obtain an explicit relation for the maximum depth drilled and hence time taken for achieving that? This research did make an attempt in that direction but couldn't conclusively obtain an explicit relation for the maximum depth. The biggest hindrance in obtaining such a relation is the very nature of the model and the mathematical expressions that we have obtained. One way to approach this problem could be to look for an altogether new model.
2. How do we incorporate mesh size mathematically into the model? Though the present research delved into the effects of mesh sizes on model and the model parameters but couldn't explicitly relate mesh size to the model mathematically.

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